

On A Generalization of Weak Armendariz Rings

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ABSTRACT: We introduce the notion of J-Armendariz rings, which are a generalization of weak Armendariz rings and investigate their properties. We show that any local ring is J-Armendariz, and then find a local ring that is not weak Armendariz. Moreover, we prove that a ring R is J-Armendariz if and only if the n -by- n upper triangular matrix ring $T_n(R)$ is J-Armendariz. For a ring R and for some $e^2 = e \in R$, we show that if R is an abelian ring, then R is J-Armendariz if and only if eRe is J-Armendariz. Also if the polynomial ring $R[x]$ is J-Armendariz, then it is proven that the Laurent polynomial ring $R[x, x^{-1}]$ is J-Armendariz.

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1 Introduction

Throughout this article, R denotes an associative ring with identity. For a ring R , $Nil(R)$ denotes the set of nilpotents elements in R . In 1997, Rege and Chhawchharia introduced the notion of an Armendariz ring. They called a ring R an Armendariz ring if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_ib_j = 0$ for all i and j . The name "Armendariz ring"

is chosen because Armendariz [2, Lemma 1] has been shown that reduced ring (that is a ring without nonzero nilpotent) satisfies this condition. A number of properties of the Armendariz rings have been studied in [1, 2, 3, 5, 7, 9]. So far Armendariz rings are generalized in several forms. A generalization of Armendariz rings has been investigated in [4] Liu and Zhao [8] called a ring R weak Armendariz if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$, $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j \in Nil(R)$ for all i and j . Recall that the Jacobson radical of a ring R , is defined to be the intersection of all the maximal left ideals of R . We use $J(R)$ to denote the Jacobson radical of R . We call a ring R , *J-Armendariz* if whenever polynomials $f(x) = a_0 + a_1x + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + \cdots + b_mx^m \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_ib_j \in J(R)$ for all i and j . Clearly, weak Armendariz rings are J-Armendariz. Moreover, for an artinian ring, weak Armendariz rings and J-Armendariz rings are the same. But, there exist a J-Armendariz ring that are not weak Armendariz. Thus J-Armendariz rings are a proper generalization of weak Armendariz rings. Furthermore, we prove that the local rings are J-Armendariz. Then we give an example to show that Local rings are not weak Armendariz in general.

2 J-Armendariz rings

In this section J-Armendariz rings are introduced as a generalization of weak Armendariz ring.

Definition 2.1. *A ring R is said to be J-Armendariz if for any nonzero polynomial $f(x) = \sum_{i=0}^n a_ix^i$ and $g(x) = \sum_{j=0}^m b_jx^j \in R[x]$, $f(x)g(x) = 0$, implies that $a_ib_j \in J(R)$ for each i, j .*

Clearly, any Armendariz ring and weak Armendariz ring is J-Armendariz. In the following, we will see that the J-Armendariz rings are not necessary weak Armendariz.

Example 2.2. *Let A be the 3 by 3 full matrix ring over the power series ring $F[[t]]$ over a field F . Let*

$$B = \{M = (m_{ij}) \in A \mid m_{ij} \in tF[[t]] \text{ for } 1 \leq i, j \leq 2 \text{ and } m_{ij} = 0 \text{ for } i = 3 \text{ or } j = 3\}$$

$$C = \{M = (m_{ij}) \in A \mid m_{ii} \in F \text{ and } m_{ij} = 0 \text{ for } i \neq j\}.$$

Let R be the subring of A generated by B and C . Let $F = \mathbb{Z}_2$. Note that every element of R is of the form $\begin{pmatrix} a+f_1 & f_2 & 0 \\ f_3 & a+f_4 & 0 \\ 0 & 0 & a \end{pmatrix}$ for some $a \in F$ and $f_i \in tF[[t]]$ ($i = 1, 2, 3, 4$) and $J(R) = tR$. Let

$$f(x) = \sum_{i=0}^n \begin{pmatrix} a_i+f_{1_i} & f_{2_i} & 0 \\ f_{3_i} & a_i+f_{4_i} & 0 \\ 0 & 0 & a_i \end{pmatrix} x^i \text{ and } g(x) = \sum_{j=0}^m \begin{pmatrix} b_j+g_{1_j} & g_{2_j} & 0 \\ g_{3_j} & b_j+g_{4_j} & 0 \\ 0 & 0 & b_j \end{pmatrix} x^j \in R[x].$$

Assume that $f(x)g(x) = 0$. Then $a_i b_j = 0$ for all i and j and so

$$\begin{pmatrix} a_i+f_{1_i} & f_{2_i} & 0 \\ f_{3_i} & a_i+f_{4_i} & 0 \\ 0 & 0 & a_i \end{pmatrix} \begin{pmatrix} b_j+g_{1_j} & g_{2_j} & 0 \\ g_{3_j} & b_j+g_{4_j} & 0 \\ 0 & 0 & b_j \end{pmatrix} \in tR.$$

Hence R is J -Armendariz. Now consider two polynomials over R

$$f(x) = te_{11} + te_{12}x + te_{21}x^2 + te_{22}x^3, \quad g(x) = -t(e_{21} + e_{22}) + t(e_{11} + e_{12})x.$$

Then $f(x)g(x) = 0$, but $te_{11}t(e_{21} + e_{22}) \notin \text{Nil}(R)$, and so the ring R is not weak Armendariz.

Proposition 2.3. *Let R be a ring and I an ideal of R such that R/I is J -Armendariz. If $I \subseteq J(R)$, then R is J -Armendariz.*

Proof. Suppose that $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m$ are polynomials in $R[x]$ such that $f(x)g(x) = 0$. This implies

$$(\bar{a}_0 + \bar{a}_1x + \bar{a}_2x^2 + \cdots + \bar{a}_nx^n)(\bar{b}_0 + \bar{b}_1x + \bar{b}_2x^2 + \cdots + \bar{b}_mx^m) = \bar{0},$$

in R/I . Thus $\bar{a}_i\bar{b}_j \in J(R/I)$, And so $a_ib_j \in J(R)$. This means that R is a J -Armendariz ring. \square

Corollary 2.4. *Let R be any local ring. Then R is J -Armendariz.*

One may ask if local rings are weak Armendariz, but the following gives a negative answer.

Example 2.5. *Let F be a field, $R = M_2(F)$ and $R_1 = R[[t]]$. Consider the ring*

$$S = \{\sum_{i=0}^{\infty} a_i t^i \in R_1 | a_0 \in kI \text{ for } k \in F\},$$

where I is the identity matrix over F . It is obvious that S is local and so is J -Armendariz. Now for $f(x) = e_{11}t - e_{12}tx$ and $g(x) = e_{21}t + e_{11}tx \in S[x]$, we have $f(x)g(x) = 0$, but $(e_{11}t)^2$ is not nilpotent in S , and so S is not weak Armendariz.

Theorem 2.6. *Let R_t be a ring, for each $t \in I$. Then any direct product of rings $\prod_{t \in I} R_t$, is J -Armendariz if and only if any R_t is J -Armendariz.*

Proof. Suppose that R_t is J -Armendariz, for each $t \in I$ and $R = \prod_{t \in I} R_t$. Let $f(x)g(x) = 0$ for some polynomials $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, $g(x) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m \in R[x]$, where $a_i = (a_{i_1}, a_{i_2}, \dots, a_{i_t}, \dots)$, $b_j = (b_{j_1}, b_{j_2}, \dots, b_{j_t}, \dots)$ are elements of the product ring R for $1 \leq i \leq n$ and $1 \leq j \leq m$. Define

$$f_t(x) = a_{0_t} + a_{1_t}x + a_{2_t}x^2 + \cdots + a_{n_t}x^n, \quad g_t(x) = b_{0_t} + b_{1_t}x + b_{2_t}x^2 + \cdots + b_{m_t}x^m.$$

From $f(x)g(x) = 0$, we have $a_0b_0 = 0, a_0b_1 + a_1b_0 = 0, \dots, a_nb_m = 0$, and this implies

$$a_{0_1}b_{0_1} = a_{0_2}b_{0_2} = \cdots = a_{0_t}b_{0_t} = \cdots = 0$$

$$a_{0_1}b_{1_1} + a_{1_1}b_{0_1} = a_{0_2}b_{1_2} + a_{1_2}b_{0_2} = \cdots = a_{0_t}b_{1_t} + a_{1_t}b_{0_t} = \cdots = 0$$

$$a_{n_1}b_{m_1} = a_{n_2}b_{m_2} = \cdots = a_{n_t}b_{m_t} = \cdots = 0$$

This means that $f_t(x)g_t(x) = 0$ in $R_t[x]$, for each $t \in I$. Since R_t is J -Armendariz for each $t \in I$, then $a_{i_t}b_{j_t} \in J(R_t)$. Now the equation $\prod_{t \in I} J(R_t) = J(\prod_{t \in I} R_t)$, implies that $a_i b_j \in J(R)$, and so R is J -Armendariz. Conversely, assume that $R = \prod_{t \in I} R_t$ is J -Armendariz and $f_t(x)g_t(x) = 0$ for some polynomials $f_t(x) = a_{0_t} + a_{1_t}x + a_{2_t}x^2 + \cdots + a_{n_t}x^n$, $g_t(x) = b_{0_t} + b_{1_t}x + b_{2_t}x^2 + \cdots + b_{m_t}x^m \in R_t[x]$, with $t \in I$. Define $F(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$, $G(x) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m \in R[x]$, where $a_i = (0, \dots, 0, a_{i_t}, 0, \dots)$, $b_j = (0, \dots, 0, b_{j_t}, 0, \dots) \in R$. Since $f_t(x)g_t(x) = 0$, we have $F(x)G(x) = 0$. R is J -Armendariz, so $a_i b_j \in J(R)$. Therefore $a_{i_t} b_{j_t} \in J(R_t)$ and so R_t is J -Armendariz for each $t \in I$. \square

The following example shows that for an Armendariz ring R , every full n -by- n matrix ring $M_n(R)$ over R need not to be J-Armendariz.

Example 2.7. Let F be a field and $R = M_2(F)$. If $f(x) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}x$ and $g(x) = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}x$, then $f(x)g(x) = 0$. But $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ is not in $J(R)$. Thus R is not J-Armendariz.

Let R and S be two rings and M be an (R, S) -bimodule. This means that M is a left R -module and a right S -module such that $(rm)s = r(ms)$ for all $r \in R$, $m \in M$, and $s \in S$. Given such a bimodule M we can form

$$T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix} = \left\{ \begin{pmatrix} r & m \\ 0 & s \end{pmatrix} : r \in R, m \in M, s \in S \right\}$$

and define a multiplication on T by using formal matrix multiplication:

$$\begin{pmatrix} r & m \\ 0 & s \end{pmatrix} \begin{pmatrix} r' & m' \\ 0 & s' \end{pmatrix} = \begin{pmatrix} rr' & rm' + ms' \\ 0 & ss' \end{pmatrix}.$$

This ring construction is called triangular ring T .

Proposition 2.8. Let R and S be two rings and T be the triangular ring $T = \begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$ (where M is an (R, S) -bimodule). Then the rings R and S are J-Armendariz if and only if T is J-Armendariz.

Proof. Let R and S be J-Armendarz, and

$$f(x) = \begin{pmatrix} r_0 & m_0 \\ 0 & s_0 \end{pmatrix} + \begin{pmatrix} r_1 & m_1 \\ 0 & s_1 \end{pmatrix}x + \cdots + \begin{pmatrix} r_n & m_n \\ 0 & s_n \end{pmatrix}x^n,$$

$$g(x) = \begin{pmatrix} r'_0 & m'_0 \\ 0 & s'_0 \end{pmatrix} + \begin{pmatrix} r'_1 & m'_1 \\ 0 & s'_1 \end{pmatrix}x + \cdots + \begin{pmatrix} r'_m & m'_m \\ 0 & s'_m \end{pmatrix}x^m \in T[x]$$

satisfy $f(x)g(x) = 0$. Define

$$f_r(x) = r_0 + r_1x + \cdots + r_nx^n, g_r(x) = r'_0 + r'_1x + \cdots + r'_mx^m \in R[x]$$

and

$$f_s(x) = s_0 + s_1x + \cdots + s_nx^n, g_s(x) = s'_0 + s'_1x + \cdots + s'_mx^m \in S[x].$$

From $f(x)g(x) = 0$, we have $f_r(x)g_r(x) = f_s(x)g_s(x) = 0$, and since R and S are J-Armendariz then $r_i r'_j \in J(R)$ and $s_i s'_j \in J(S)$ for each $1 \leq i \leq n$, $1 \leq j \leq m$. Now from the fact $J(T) = \begin{pmatrix} J(R) & M \\ 0 & J(S) \end{pmatrix}$, we obtain that $\begin{pmatrix} r_i & m_i \\ 0 & s_i \end{pmatrix} \begin{pmatrix} r'_j & m'_j \\ 0 & s'_j \end{pmatrix} \in J(T)$ for any i, j . Hence T is a J-Armendariz ring. Conversely, let T be a J-Armendariz ring, $f_r(x) = r_0 + r_1 x + \cdots + r_n x^n$, $g_r(x) = r'_0 + r'_1 x + \cdots + r'_m x^m \in R[x]$, such that $f_r(x)g_r(x) = 0$, and $f_s(x) = s_0 + s_1 x + \cdots + s_n x^n$, $g_s(x) = s'_0 + s'_1 x + \cdots + s'_m x^m \in S[x]$, such that $f_s(x)g_s(x) = 0$. If

$$\begin{aligned} f(x) &= \begin{pmatrix} r_0 & 0 \\ 0 & s_0 \end{pmatrix} + \begin{pmatrix} r_1 & 0 \\ 0 & s_1 \end{pmatrix} x + \cdots + \begin{pmatrix} r_n & 0 \\ 0 & s_n \end{pmatrix} x^n \text{ and} \\ g(x) &= \begin{pmatrix} r'_0 & 0 \\ 0 & s'_0 \end{pmatrix} + \begin{pmatrix} r'_1 & 0 \\ 0 & s'_1 \end{pmatrix} x + \cdots + \begin{pmatrix} r'_m & 0 \\ 0 & s'_m \end{pmatrix} x^m \in T[x] \end{aligned}$$

Then from $f_r(x)g_r(x) = 0$ and $f_s(x)g_s(x) = 0$ it follows that $f(x)g(x) = 0$. Since T is a J-Armendariz ring, $\begin{pmatrix} r_i & 0 \\ 0 & s_i \end{pmatrix} \begin{pmatrix} r'_j & 0 \\ 0 & s'_j \end{pmatrix} \in J(T) = \begin{pmatrix} J(R) & 0 \\ 0 & J(S) \end{pmatrix}$. Thus $r_i r'_j \in J(R)$ and $s_i s'_j \in J(S)$ for any i, j . This shows that R and S are J-Armendariz. \square

Given a ring R and a bimodule ${}_R M_R$, the trivial extension of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and the multiplication

$$(r_1, m_1)(r_2, m_2) = (r_1 r_2, r_1 m_2 + m_1 r_2).$$

This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

Corollary 2.9. *A ring R is J-Armendariz if and only if the trivial extension $T(R, R)$ is a J-Armendariz ring.*

Corollary 2.10. *A ring R is J-Armendariz if and only if, for any n , $T_n(R)$ is J-Armendariz.*

Corollary 2.11. *If R is a Armendariz ring then, for any n , $T_n(R)$ is a J-Armendariz ring.*

Recall that a ring R is said to be *abelian* if every idempotent of it is central. Armendariz rings are abelian [7, Lemma 7], but the next example shows that weak Armendariz

and J-Armendariz rings need not to be abelian in general.

Example 2.12. Let F be a field. By Corollary 2.11, $R = T_2(F)$ is a J-Armendariz ring. We see that $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is an idempotent element in R , that is not central. So R is not an abelian ring.

Proposition 2.13. Let R be a J-Armendariz ring. Then for any idempotent e of R , eRe is J-Armendariz. The converse holds if R is an abelian ring.

Proof. Let $f(x) = \sum_{i=0}^n a_i x^i$, $g(x) = \sum_{j=0}^m b_j x^j \in (eRe)[x]$ be such that $f(x)g(x) = 0$. Since R is J-Armendariz and $a_i, b_j \in eRe \subseteq R$, then we have $a_i b_j \in J(R) \cap eRe = J(eRe)$. This means that eRe is J-Armendariz. Conversely, let eRe be a J-Armendariz ring and $f(x) = \sum_{i=0}^n a_i x^i$, $g(x) = \sum_{j=0}^m b_j x^j \in R[x]$, such that $f(x)g(x) = 0$. By the hypothesis, $0 = ef(x)eg(x)e \in (eRe)[x]$, and since eRe is J-Armendariz, we have $a_i b_j \in J(eRe) = J(R) \cap eRe$. Thus R is J-Armendariz. \square

In [1] it is proven that a ring R is Armendariz if and only if its polynomial ring $R[x]$ is Armendariz. More generally, we can get the following result.

Theorem 2.14. If the ring $R[x]$ is J-Armendariz, then R is J-Armendariz. The converse holds if $J(R)[x] \subseteq J(R[x])$.

Proof. Suppose that $R[x]$ is a J-Armendariz ring. Let $f(y) = \sum_{i=0}^n a_i y^i$ and $g(y) = \sum_{j=0}^m b_j y^j$ be nonzero polynomials $\in R[y]$, such that $f(y)g(y) = 0$. Since $R[x]$ is J-Armendariz and $R \subseteq R[x]$, we have $a_i b_j \in R \cap J(R[x])$, and so R is J-Armendariz. Conversely, suppose that R is J-Armendariz and $J(R)[x] \subseteq J(R[x])$. Let $F(y) = f_0 + f_1 y + \cdots + f_n y^n$ and $G(y) = g_0 + g_1 y + \cdots + g_m y^m$ be polynomials in $R[x][y]$, with $F(y)G(y) = 0$. We also let $f_i(x) = a_{i0} + a_{i1}x + a_{i2}x^2 + \cdots + a_{i\omega_i}x^{\omega_i}$ and $g_j(x) = b_{j0} + b_{j1}x + b_{j2}x^2 + \cdots + b_{j\nu_j}x^{\nu_j} \in R[x]$ for each $0 \leq i \leq n$ and $0 \leq j \leq m$. Take a positive integer t such that $t \geq \deg(f_0(x)) + \deg(f_1(x)) + \cdots + \deg(f_n(x)) + \deg(g_0(x)) + \deg(g_1(x)) + \cdots + \deg(g_m(x))$, where the degree is as polynomials in x and the degree of zero polynomial is taken to be 0. Then $F(x^t) = f_0 + f_1 x^t + \cdots + f_n x^{tn}$ and $G(x^t) =$

$g_0 + g_1x^t + \cdots + g_mx^{tm} \in R[x]$ and the set of coefficients of the f_i 's (resp. g_j 's) equals the set of coefficients of the $F(x^t)$ (resp. $G(x^t)$). Since $F(y)G(y) = 0$, then $F(x^t)G(x^t) = 0$. So $a_{is_i}b_{jr_j} \in J(R)$, where $0 \leq s_i \leq \omega_i$, $0 \leq r_j \leq \nu_j$. By hypothesis we have $J(R)[x] \subseteq J(R[x])$, and so $f_i g_j \in J(R[x])$. It implies that R is J-Armendariz. \square

Proposition 2.15. *Let R is a J-Armendariz ring and S denotes a multiplicatively closed subset of a ring R consisting of central regular elements. Let $S^{-1}R$ denotes the localization of R at S . Then $S^{-1}R$ is a J-Armendariz ring.*

Proof. Suppose that R is a J-Armendariz ring. Let $F(x) = \sum_{i=0}^n (\alpha_i)x^i$ and $G(x) = \sum_{j=0}^m (\beta_j)x^j$ be nonzero polynomials in $(S^{-1}R)[x]$ such that $F(x)G(x) = 0$, where $\alpha_i = a_i u^{-1}$, $\beta_j = b_j v^{-1}$, with $a_i, b_j \in R$ and $u, v \in S$. Since S is contained in the center of R , we have $F(x)G(x) = (uv)^{-1}(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n)(b_0 + b_1x + b_2x^2 + \cdots + b_mx^m) = 0$. Let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ and $g(x) = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m$. Then $f(x)$ and $g(x)$ are nonzero polynomials in $R[x]$ with $f(x)g(x) = 0$. Since R is J-Armendariz, then $a_i b_j \in J(R)$. It means that $\alpha_i \beta_j \in J(S^{-1}R)$, concluding that $S^{-1}R$ is J-Armendariz. \square

Corollary 2.16. *For a ring R , if $R[x]$ is J-Armendariz, Then $R[x, x^{-1}]$ is J-Armendariz.*

Proof. Let $S = \{1, x, x^2, x^3, x^4, \dots\}$. Then S is a multiplicatively closed subset of $R[x]$ consisting of central regular elements. Then the proof follows from Proposition 2.15. \square

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